

IFUM-732-FT
hep-th/0212096

AdS₂ Supergravity and Superconformal Quantum Mechanics

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Abstract

We investigate the asymptotic dynamics of topological anti-de Sitter supergravity in two dimensions. Starting from the formulation as a BF theory, it is shown that the AdS₂ boundary conditions imply that the asymptotic symmetries form a super-Virasoro algebra. Using the central charge of this algebra in Cardy's formula, we exactly reproduce the thermodynamical entropy of AdS₂ black holes. Furthermore, we show that the dynamics of the dilaton and its superpartner reduces to that of superconformal transformations that leave invariant one chiral component of the stress tensor supercurrent of a two-dimensional conformal field theory. This dynamics is governed by a supersymmetric extension of the de Alfaro-Fubini-Furlan model of conformal quantum mechanics. Finally, two-dimensional de Sitter gravity is also considered, and the dS₂ entropy is computed by counting CFT states.

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1 Introduction

There is by now a great deal of evidence that the correspondence between type IIB string theory on $\text{AdS}_5 \times S^5$ and $\mathcal{N} = 4$ super-Yang-Mills theory in four dimensions [1] extends to a more general duality between any gravitational theory on anti-de Sitter spaces and conformal field theories residing on the boundary of AdS [2]. Such a duality represents an explicit realization of the holographic principle [3]. In three dimensions, pure Einstein-AdS gravity was known to be related to a two-dimensional conformal field theory [4] even before the advent of the AdS/CFT correspondence. This is based on the fact that pure Einstein gravity in three dimensions can be written as a Chern-Simons theory [5, 6], which is known to reduce to a WZNW model in presence of a boundary [7]. The boundary conditions for asymptotically AdS spaces [8] provide then the constraints for a Hamiltonian reduction from the WZNW model to Liouville field theory [4]¹. Recently the equivalence of pure gravity in three dimensions and two-dimensional Liouville theory has been extended in [10, 11] to the case of positive cosmological constant, providing thus an explicit example of the proposed correspondence between de Sitter gravity and Euclidean conformal field theories [12–14]².

Much less is known for two-dimensional anti-de Sitter gravity [16], which, in the spirit of AdS/CFT, should have a dual description in terms of a conformal quantum mechanical system [17]³. Progress in this direction has been made in [19], where it was shown that two-dimensional Jackiw-Teitelboim-AdS gravity [20] induces on the spacetime boundary a conformally invariant dynamics that can be described in terms of a de Alfaro-Fubini-Furlan (DFF) model [21] coupled to an external source⁴.

In the present paper we will extend the results of [19, 22] to topological AdS_2 supergravity. Making use of its gauge theory formulation [23, 24], we will be able to show in a clear and transparent way how the superconformal algebra appears in this system, and how the dynamics can be reduced to that of a superconformally invariant quantum mechanics.

The remainder of this paper is organized as follows: In section 2, the formulation of topological AdS_2 supergravity as a gauge theory (BF theory) is briefly reviewed. In section 3, we translate the boundary conditions for asymptotically AdS_2 spaces [25, 26] into conditions for the bosonic fields appearing in the gauge theory. Furthermore, suitable boundary conditions on the fermions are defined. Subsequently, we show that the gauge transformations preserving this asymptotic behaviour generate a super-Virasoro algebra, and compute its central charge. Using this in Cardy’s formula for the asymptotic level density, we exactly reproduce the thermodynamical entropy of AdS_2 black holes. In the following section, it is shown that the dynamics of the dilaton and its superpartner reduces to that of superconformal transformations that leave invariant one chiral component of the stress tensor supercurrent of a two-dimensional conformal field theory. This dynamics turns out to be governed by a supersymmetric extension of the DFF model of conformal

¹The results of [4] have been generalized to three-dimensional extended AdS supergravity in [9].

²Cf. [15] for earlier work.

³Cf. also [18] and references therein.

⁴Cf. [22] for related work.

quantum mechanics. In section 5, we generalize the considerations of sections 3-4, which refer to AdS gravity, to the de Sitter case. Again, a microstate counting yields precisely the entropy of two-dimensional de Sitter space. We close our paper with some final remarks, followed by an appendix that contains our conventions.

2 Two-dimensional anti-de Sitter supergravity as topological gauge theory

We consider the minimal supersymmetric extension of the Jackiw-Teitelboim (JT) model [20], which contains the zweibein e_μ^a , a Rarita-Schwinger field $\psi_{i\mu}$, the dilaton η and dilatino λ_i . The action is given by [27]

$$I = \frac{\beta}{2} \int d^2x \sqrt{-g} \eta (R - 2\Lambda) + \frac{\beta}{4l} \int \eta \psi \wedge \gamma_5 \bar{\psi} + \frac{\beta}{4} \int \bar{\lambda} D\psi, \quad (2.1)$$

where β denotes a dimensionless coupling constant, $\Lambda = 1/l^2$ is the cosmological constant⁵ and

$$(D\psi)_j = d\psi_j - \frac{1}{4}(\gamma_{ab})^k{}_j \omega^{ab} \wedge \psi_k + \frac{i}{2l}(\gamma_a)^k{}_j e^a \wedge \psi_k. \quad (2.2)$$

$\psi_{i\mu}$ and λ_i are Majorana spinors, i. e. $\lambda_i = \epsilon_{ij} \bar{\lambda}^i$ etc., where $\epsilon_{12} = 1$.

The scalar curvature R is calculated using the spin connection that solves the torsion-free condition

$$T^a = de^a + \omega^a{}_b \wedge e^b + \frac{i}{4}(\gamma^a)^j{}_k \psi^k \wedge \psi_j = 0. \quad (2.3)$$

Notice that, throughout the paper, the product of two fermions differs by a factor i from the standard Grassmann product fulfilling $(ab)^* = b^*a^*$.

The supergravity model (2.1) can be formulated as an $\text{Osp}(1|2)$ topological gauge theory [24]. This is similar to the three-dimensional case, where the Einstein-AdS supergravity action can be written as a sum of two Chern-Simons actions [5]. Let us introduce the superconnection

$$\Gamma = A^A \tau_A + \psi_i R^i \equiv A + \psi, \quad (2.4)$$

and the scalar multiplet

$$S = \phi^A \tau_A + \lambda_i R^i \equiv \phi + \lambda \quad (2.5)$$

⁵Notice that for the $(+, -)$ signature used here, Λ is positive for AdS and negative for dS.

in the adjoint representation of $\text{Osp}(1|2)$. In (2.4) and (2.5), the τ_A and R^i are respectively the bosonic and fermionic generators of the graded Lie algebra $\text{osp}(1|2)$ (cf. appendix A.1). The action describing AdS_2 supergravity is given by⁶

$$I = \frac{\beta}{2} \int \text{STr}(S\mathcal{F}), \quad (2.6)$$

where

$$\mathcal{F} = d\Gamma + \Gamma \wedge \Gamma \quad (2.7)$$

denotes the $\text{Osp}(1|2)$ field strength.

The action (2.6) is invariant under the gauge transformations

$$\delta_{\Xi}\Gamma = d\Xi + [\Gamma, \Xi], \quad (2.8)$$

$$\delta_{\Xi}S = [S, \Xi]. \quad (2.9)$$

The equations of motion following from (2.6) read

$$\mathcal{F} = 0, \quad (2.10)$$

$$DS \equiv dS + [\Gamma, S] = 0, \quad (2.11)$$

so that the solutions are those of flat $\text{Osp}(1|2)$ connections in two dimensions.

To make contact with gravity, one decomposes the fields A and ϕ according to

$$A^A = (l^{-1}e^a, \omega), \quad \phi^A = (\phi^a, 4\eta), \quad (2.12)$$

where $\omega = \omega^0_1$ and $a = 0, 1$. Using (2.12), the bosonic part $F \equiv dA + A \wedge A + \psi \wedge \psi$ of the field strength (2.7) reads

$$F^A = (l^{-1}T^a, d\omega + l^{-2}e^0 \wedge e^1 - \frac{1}{4}\psi \wedge \gamma_5 \bar{\psi}), \quad (2.13)$$

where T^a will correspond to the torsion tensor given in (2.3). Inserting (2.13) into the action (2.6), we see that vanishing torsion is enforced by the Lagrange multipliers ϕ^a . Eliminating the ϕ^a by going to the second order formalism, and rescaling $\psi \rightarrow \psi/\sqrt{l}$, $\lambda \rightarrow \lambda\sqrt{l}$, we recover (2.1)⁷.

⁶(2.6) can also be obtained from the super-Chern-Simons action $I = \frac{k}{4\pi} \int \text{STr}(\hat{\Gamma} \wedge d\hat{\Gamma} + \frac{2}{3}\hat{\Gamma} \wedge \hat{\Gamma} \wedge \hat{\Gamma})$ using the dimensional reduction ansatz $\hat{\Gamma} = \Gamma + Sdz$.

⁷The formulation of the supergravity theory defined by the action (2.1) as a gauge theory involves the identification of non-abelian gauge transformations with gravitational symmetries. For subtleties concerning this point cf. [28].

3 Asymptotic symmetries

3.1 Boundary conditions

An asymptotically AdS₂ geometry is one for which the bosonic fields A and ϕ behave for $r \rightarrow \infty$ as [22]

$$\begin{aligned} A &= \left[\frac{r}{l^2} - \frac{T_B(t)}{2\eta_0 r} \right] \tau_0 dt + \left[\frac{r}{l^2} + \frac{T_B(t)}{2\eta_0 r} \right] \tau_2 dt + \frac{dr}{r} \tau_1, \\ \phi &= \left[\alpha(t) \frac{r}{l^2} + \frac{\beta(t)}{r} \right] \tau_0 + \left[\alpha(t) \frac{r}{l^2} - \frac{\beta(t)}{r} \right] \tau_2 + \gamma(t) \tau_1, \end{aligned} \quad (3.1)$$

where the functions $T_B(t), \alpha(t), \beta(t), \gamma(t)$, that depend only on the boundary coordinate t , parametrize the subleading behaviour. η_0 is a constant to be specified below. As we will see later, $T_B(t)$ plays the role of the bosonic part of the energy-momentum tensor of a two-dimensional conformal field theory, hence the notation.

Actually the asymptotics (3.1) seems slightly more restrictive than that given in [22], which corresponds to the boundary conditions defined in [25, 26] in terms of the gravitational fields. However, it will turn out that (3.1) is still weak enough to allow for an asymptotic symmetry algebra larger than $\text{SL}(2, \mathbb{R})$. In other words, (3.1) is, up to trivial gauge transformations, the most general asymptotically AdS₂ solution. If we translate (3.1) in terms of the metric, we get

$$ds^2 = \left(\frac{r}{l} - \frac{T_B(t)l}{2\eta_0 r} \right)^2 dt^2 - \frac{l^2}{r^2} dr^2, \quad (3.2)$$

which (for zero fermions) is an *exact* solution of the equations of motion for arbitrary $T_B(t)$ ⁸. This means that (3.2) is locally isometric to AdS₂; however, solutions with different $T_B(t)$ are in general not globally equivalent.

Consider now the bosonic background of the supergravity theory (2.1) studied in [30, 31],

$$ds^2 = \left(\frac{r^2}{l^2} - a^2 \right) dt^2 - \left(\frac{r^2}{l^2} - a^2 \right)^{-1} dr^2, \quad (3.3)$$

$$\eta = \eta_0 \frac{r}{l}, \quad (3.4)$$

where a and $\eta_0 \geq 0$ denote integration constants. Following Ref. [31], the solutions with a^2 positive, negative or zero will be denoted by AdS₂⁺, AdS₂[−] and AdS₂⁰ respectively. Note that due to the presence of the dilaton, which represents a position-dependent coupling constant, the solutions with different sign of a^2 are physically inequivalent [31]. In particular, the solution with positive a^2 represents a black hole with event horizon at $r = al$. The black hole mass and Bekenstein-Hawking entropy are respectively given by [31]

⁸This is similar to the three-dimensional case, where two arbitrary functions appear [29].

$$M = \frac{1}{2l}\eta_0 a^2, \quad S = 2\pi\eta_0 a. \quad (3.5)$$

Applying the diffeomorphism $r \mapsto r + a^2 l^2 / 4r$ to the metric (3.3) yields

$$ds^2 = \left(\frac{r}{l} - \frac{a^2 l}{4r} \right)^2 dt^2 - \frac{l^2}{r^2} dr^2. \quad (3.6)$$

Comparing this with (3.2), we see that in this case one has $T_B(t) = \eta_0 a^2 / 2 = Ml$, so it essentially coincides with the black hole mass.

The boundary conditions (3.1) must of course be completed by giving the asymptotic behaviour of the fermions, which can be obtained by following the general scheme introduced in [32, 33]. Namely, one takes a generic configuration satisfying the above asymptotic boundary conditions with vanishing fermions, and acts on it with the exact symmetries of AdS_2^0 . This will create new terms, from which one can infer the desired asymptotic behaviour.

In order to determine the exact fermionic symmetries of AdS_2^0 , one solves the Killing spinor equation

$$\delta_\epsilon \psi = d\epsilon + [A, \epsilon] = 0, \quad (3.7)$$

with the connection A given by (3.1) with $T_B(t) = 0$. The solution $\epsilon = \epsilon_1 R^1 + \epsilon_2 R^2$ of (3.7) reads

$$\epsilon_{1,2} = \sqrt{\frac{l}{r}} \left(1 - \frac{rt}{l^2} \right) \epsilon_0^{(1)} \pm \sqrt{\frac{r}{l}} \epsilon_0^{(2)}, \quad (3.8)$$

where $\epsilon_0^{(1,2)}$ denote two arbitrary constant spinors.

Notice that the variation of the dilatino under supersymmetry transformations generated by ϵ is given (for a bosonic background with $\lambda = 0$) by

$$\delta_\epsilon \lambda = [\phi, \epsilon]. \quad (3.9)$$

The scalar field ϕ corresponding to AdS_2^0 is

$$\phi = 4\eta_0 \frac{r}{l} (\tau_0 + \tau_2), \quad (3.10)$$

i. e. $\alpha(t) = 4\eta_0 l$, $\beta(t) = \gamma(t) = 0$. Using (3.10), one gets

$$[\phi, \epsilon] = 4\eta_0 \sqrt{\frac{r}{l}} \left(1 - \frac{rt}{l^2}\right) \epsilon_0^{(1)} (R^1 - R^2), \quad (3.11)$$

so it vanishes only for $\epsilon_0^{(1)} = 0$. This means that the non-zero scalar field ϕ (3.10) breaks half of the supersymmetries. This is the supersymmetric analogue of the observation made in [34] that a nonvanishing field ϕ breaks the $\text{SL}(2, \mathbb{R})$ isometry group of AdS_2 down to the subgroup of time translations. We thus encounter a spontaneous partial supersymmetry breaking: The theory has $\text{osp}(1|2)$ invariance, but the vacuum preserves only an $\mathfrak{s}(2)$ subalgebra⁹.

After this small digression, we come back to the determination of the boundary conditions for the fermions. Acting on a generic background (3.1) with the exact AdS_2^0 supersymmetries (3.8), one obtains for the gravitino variation (3.7)

$$\delta_\epsilon \psi = \frac{T_B(t)}{2\eta_0 \sqrt{rl}} \epsilon_0^{(2)} (R^1 + R^2) dt, \quad (3.12)$$

so that we will impose

$$\psi = \frac{T_F(t)}{2\eta_0 \sqrt{2rl}} (R^1 + R^2) dt \quad (3.13)$$

as the appropriate boundary condition on the gravitinos, where $T_F(t)$ is an arbitrary function. Below we will see that $T_F(t)$ is the fermionic superpartner of the bosonic stress tensor $T_B(t)$.

For a generic field ϕ given in (3.1), Eq. (3.9) yields a dilatino asymptotics of the form

$$\lambda = \left(\mu(t) \sqrt{\frac{2r}{l}} + \nu(t) \sqrt{\frac{2l}{r}} \right) R^1 + \left(-\mu(t) \sqrt{\frac{2r}{l}} + \nu(t) \sqrt{\frac{2l}{r}} \right) R^2, \quad (3.14)$$

with $\mu(t)$ and $\nu(t)$ some functions.

Notice that the field strength \mathcal{F} (2.7) still vanishes for an arbitrary function $T_F(t)$ in (3.13) and for any $T_B(t)$ in (3.1), so that we have found an exact solution that contains two undetermined functions $T_{B,F}(t)$. Note also that the functions $\alpha(t)$, $\beta(t)$, $\gamma(t)$, $\mu(t)$ and $\nu(t)$ that occur in ϕ (3.1) and λ (3.14) are not arbitrary; imposing the equations of motion (2.11) yields a dynamics for these fields. In section 4, it will be shown that the resulting dynamics is that of superconformal mechanics.

⁹ $\mathfrak{s}(2)$ denotes the superalgebra introduced by Witten to formulate supersymmetric quantum mechanics [35].

3.2 Asymptotic symmetries

Having now at hand the boundary conditions for both the bosons (3.1) and fermions (3.13), (3.14), we can determine the asymptotic symmetries, i. e. the gauge transformations that preserve these conditions. It will turn out that the asymptotic symmetries form a super-Virasoro algebra.

Acting on the superconnection Γ with an infinitesimal gauge transformation $\Xi = \Lambda^A \tau_A + \epsilon_i R^i$, we get

$$\begin{aligned}
\delta_\Xi \Gamma &= d\Lambda^A \tau_A + d\epsilon_i R^i \\
&+ \left(\frac{r}{l^2} + \frac{T_B(t)}{2\eta_0 r} \right) (\Lambda^1 \tau_0 + \Lambda^0 \tau_1 + \frac{1}{2} \epsilon_1 R^1 - \frac{1}{2} \epsilon_2 R^2) dt \\
&+ \left(\frac{r}{l^2} - \frac{T_B(t)}{2\eta_0 r} \right) (-\Lambda^2 \tau_1 + \Lambda^1 \tau_2 + \frac{1}{2} \epsilon_2 R^1 - \frac{1}{2} \epsilon_1 R^2) dt \\
&+ \frac{T_F(t)}{4\sqrt{2}rl\eta_0} [(\epsilon_1 + \epsilon_2)(\tau_2 - \tau_0) - (\epsilon_1 - \epsilon_2)\tau_1 - (\Lambda^0 + \Lambda^2)(R^1 - R^2) - \Lambda^1(R^1 + R^2)] dt \\
&+ (-\Lambda^2 \tau_0 - \Lambda^0 \tau_2 + \frac{1}{2} \epsilon_2 R^1 + \frac{1}{2} \epsilon_1 R^2) \frac{dr}{r}.
\end{aligned} \tag{3.15}$$

In order to preserve the asymptotic form of the superconnection, the parameters Λ^A and ϵ_i of the gauge transformation have to be of the form

$$\begin{aligned}
\Lambda^0 &= \left(\frac{r}{l^2} - \frac{T_B(t)}{2\eta_0 r} \right) \chi(t) + \frac{l^2}{2r} \chi''(t) + \frac{l}{4\sqrt{2}\eta_0 r} T_F(t) \epsilon_0(t), \\
\Lambda^1 &= -\chi'(t), \\
\Lambda^2 &= \left(\frac{r}{l^2} + \frac{T_B(t)}{2\eta_0 r} \right) \chi(t) - \frac{l^2}{2r} \chi''(t) - \frac{l}{4\sqrt{2}\eta_0 r} T_F(t) \epsilon_0(t), \\
\epsilon_1 &= \sqrt{\frac{2r}{l}} \epsilon_0(t) + \sqrt{\frac{2l}{r}} \left(\frac{T_F(t)}{4\eta_0 l} \chi(t) - l \epsilon_0'(t) \right), \\
\epsilon_2 &= -\sqrt{\frac{2r}{l}} \epsilon_0(t) + \sqrt{\frac{2l}{r}} \left(\frac{T_F(t)}{4\eta_0 l} \chi(t) - l \epsilon_0'(t) \right),
\end{aligned} \tag{3.16}$$

where $\chi(t)$ and $\epsilon_0(t)$ denote arbitrary bosonic and fermionic functions respectively. This means that the asymptotic symmetry algebra is in fact infinite-dimensional.

Under a gauge transformation with the parameters (3.16), the fields $T_B(t)$ and $T_F(t)$ transform as

$$\delta T_B = T'_B \chi + 2T_B \chi' - \frac{c}{12} l^2 \chi''' - \frac{l}{2} T'_F \epsilon_0 - \frac{3}{2} l T_F \epsilon_0', \tag{3.17}$$

$$\delta T_F = T'_F \chi + \frac{3}{2} T_F \chi' + 2T_B \epsilon_0 - \frac{c}{3} l^2 \epsilon_0'', \tag{3.18}$$

with

$$c = 12\eta_0. \quad (3.19)$$

Eqns. (3.17) and (3.18) are exactly the transformation laws of a CFT stress tensor T_B and a spin 3/2 supercurrent T_F under superconformal transformations, with a central charge given by (3.19)¹⁰. This can be easily verified by using the superconformal Ward identities and the OPE¹¹

$$\begin{aligned} T_B(z)T_B(0) &\sim \frac{c}{2z^4} + \frac{2}{z^2}T_B(0) + \frac{1}{z}\partial T_B(0), \\ T_B(z)T_F(0) &\sim \frac{3}{2z^2}T_F(0) + \frac{1}{z}\partial T_F(0), \\ T_F(z)T_F(0) &\sim \frac{2c}{3z^3} + \frac{2}{z}T_B(0). \end{aligned} \quad (3.20)$$

The transformation rules of the fields $\alpha(t)$, $\beta(t)$, $\gamma(t)$, $\mu(t)$ and $\nu(t)$ appearing in the scalar S , can be readily obtained from (2.9), but will not be reported here.

Note that the central charge (3.19) coincides with the one computed in [36, 19, 37] for the bosonic case by different methods. This means that the central charge for AdS₂ dilaton supergravity is the same as the one for pure dilaton gravity, similar to the three-dimensional case [33].

It is instructive to translate the gauge transformations (3.16) in terms of gravitational symmetries. To this end, we consider for a moment only the bosonic case. On shell, a diffeomorphism $\delta x^\mu = v^\mu(x)$ is equivalent to a gauge transformation with parameter $\Lambda = v^\mu A_\mu$ [23]. Thus, the asymptotic symmetries (3.16) correspond to diffeomorphisms

$$\begin{aligned} \delta t &= \chi + \frac{l^2 \chi''}{\frac{2r^2}{l^2} - \frac{T_B(t)}{\eta_0}}, \\ \delta r &= -\chi' r, \end{aligned} \quad (3.21)$$

that leave the form of the metric (3.2) invariant, with $T_B \rightarrow T_B + \delta T_B$, and δT_B given by (3.17) with $\epsilon_0(t) = 0$.

3.3 Statistical entropy of AdS₂ black holes

In the preceding subsection we found that the asymptotic symmetries of AdS₂ form a superconformal algebra, with central charge (3.19). AdS₂ quantum gravity is thus a

¹⁰By introducing the dimensionless time variable $\tau = t/l$, and replacing χ by $l\chi$, (3.17) and (3.18) assume the usual form, that does not involve the length scale l .

¹¹Note that the resulting transformation laws are those of active transformations, whereas (3.17) and (3.18) are passive, so that some signs have to be changed.

holomorphic sector of a superconformal field theory. The asymptotic growth of the number of states is given by the Cardy formula [38]

$$S = 2\pi\sqrt{\frac{cL_0}{6}}. \quad (3.22)$$

Using $L_0 = Ml$, with the black hole mass M given in (3.5), and the central charge (3.19), one exactly reproduces the thermodynamical entropy (3.5) of the AdS_2 black hole.

Similar to the case of three dimensions [39, 40], we are led to identify the zero mass black hole AdS_2^0 ($a^2 = 0$ in (3.3)) with the Ramond ground state of the superconformal field theory, whereas AdS_2^- with $a^2 = -1$ (which is AdS_2 in global coordinates) corresponds to the Neveu-Schwarz ground state. The NS ground state has a mass shift

$$L_0 = -\frac{c}{24} = -\frac{1}{2}\eta_0,$$

so that $M = L_0/l$ coincides exactly with the mass (3.5) of AdS_2^- for $a^2 = -1$. Further evidence for this identification comes from the fact that the covariantly constant spinors for AdS_2^- ,

$$\begin{aligned} \epsilon_1 &= \left(\epsilon_0^{(1)} \cos \frac{t}{2l} + \epsilon_0^{(2)} \sin \frac{t}{2l} \right) \cosh \frac{\zeta}{2} + \left(-\epsilon_0^{(1)} \sin \frac{t}{2l} + \epsilon_0^{(2)} \cos \frac{t}{2l} \right) \sinh \frac{\zeta}{2}, \\ \epsilon_2 &= -\left(\epsilon_0^{(1)} \cos \frac{t}{2l} + \epsilon_0^{(2)} \sin \frac{t}{2l} \right) \sinh \frac{\zeta}{2} + \left(\epsilon_0^{(1)} \sin \frac{t}{2l} - \epsilon_0^{(2)} \cos \frac{t}{2l} \right) \cosh \frac{\zeta}{2}, \end{aligned}$$

where $\epsilon_0^{(1,2)}$ denote arbitrary constant spinors, and $\sinh \zeta \equiv r/l$, are antiperiodic under $t \rightarrow t + 2\pi l$.

Notice in this context that the coordinate transformation

$$\begin{aligned} t &\rightarrow ie^{it/l} \frac{r}{\sqrt{1 + \frac{r^2}{l^2}}}, \\ r &\rightarrow le^{-it/l} \sqrt{1 + \frac{r^2}{l^2}}, \end{aligned} \quad (3.23)$$

maps the AdS_2^0 metric (the Ramond ground state) to the AdS_2^- metric with $a^2 = -1$, i. e. , to the Neveu-Schwarz ground state. But near the boundary $r \rightarrow \infty$, Eq. (3.23) becomes precisely the map from the line to the circle with circumference $2\pi l$.

4 Reduction to superconformal quantum mechanics

As we mentioned above, the functions $\alpha(t)$, $\beta(t)$, $\gamma(t)$, $\mu(t)$ and $\nu(t)$ that appear in ϕ (3.1) and λ (3.14) are not arbitrary; imposing the equations of motion (2.11) one obtains

$$T'_B\alpha + 2T_B\alpha' - \frac{c}{12}l^2\alpha''' - \frac{l}{2}T'_F\mu - \frac{3}{2}lT_F\mu' = 0, \quad (4.1)$$

$$T'_F\alpha + \frac{3}{2}T_F\alpha' + 2T_B\mu - \frac{c}{3}l^2\mu'' = 0, \quad (4.2)$$

and

$$\begin{aligned} \frac{c}{6}\beta &= -T_B\alpha + \frac{l}{2}T_F\mu + \frac{c}{12}l^2\alpha'', \\ \frac{c}{3}l\nu &= T_F\alpha - \frac{c}{3}l^2\mu', \\ \gamma &= -\alpha'. \end{aligned} \quad (4.3)$$

For given stress tensor $T_B(t)$ and spin 3/2 current $T_F(t)$, the equations (4.1), (4.2) determine $\alpha(t)$ and $\mu(t)$. From the remaining relations (4.3) one obtains then the functions $\beta(t)$, $\gamma(t)$ and $\nu(t)$. Comparing (4.1) and (4.2) with (3.17) and (3.18), we see that $\alpha(t)$ and $\mu(t)$ correspond to superconformal transformations that leave the stress tensor and its superpartner invariant. Since Eqns. (4.1), (4.2) result from the equation of motion (2.11) for the scalar multiplet S , this is not surprising: S generates gauge transformations (2.8) that do not change the superconnection Γ .

What is less obvious, and comes as a surprise, is that the dynamics described by (4.1) and (4.2) is precisely that of superconformal mechanics. To show this, we multiply Eq. (4.1) by $-12\alpha/cl^2$, and add this to (4.2), multiplied (from the right) by $12\mu/cl$, to obtain

$$\left(\alpha\alpha'' - \frac{1}{2}\alpha'^2 - \frac{12}{cl^2}T_B\alpha^2 + \frac{18}{cl}T_F\alpha\mu - 4l\mu'\mu \right)' = 0. \quad (4.4)$$

Defining

$$\alpha =: q^2, \quad (4.5)$$

one gets from integration of (4.4)

$$q'' - \frac{g}{q^3} - \frac{6}{cl^2}T_Bq - \frac{2l\mu'\mu}{q^3} + \frac{9}{clq}T_F\mu = 0, \quad (4.6)$$

where g denotes an integration constant. Next, we multiply Eq. (4.2) by q , and add this to (4.6), multiplied by $cl^2\mu/3$. This yields

$$\varsigma^1 = -\frac{q^2}{\sqrt{g}}\varsigma^{2'}, \quad (4.7)$$

where the spinors $\varsigma^{1,2}$ are defined by

$$\varsigma^1 := \frac{l}{q}\mu, \quad (4.8)$$

$$\varsigma^2 := \frac{q^2}{\sqrt{g}}\varsigma^{1'} - \frac{3T_F q^3}{cl\sqrt{g}}. \quad (4.9)$$

Using the definitions of $\varsigma^{1,2}$ in (4.6), one finally obtains

$$q'' - \frac{g}{q^3} - \frac{6}{cl^2}T_B q + \frac{\sqrt{g}}{lq^3}[\varsigma^1, \varsigma^2] + \frac{3}{cl^2}T_F \varsigma^1 = 0. \quad (4.10)$$

(4.7), (4.9) and (4.10) are precisely the equations of motion of the $\text{osp}(2|2)$ superconformal quantum mechanics considered in [41, 42] (the $\mathcal{N} = 2$ superextension of the DFF model [21]), with an additional coupling of the fields $q(t)$ and $\varsigma^1(t)$ to the external sources $T_B(t)$ and $T_F(t)$. These equations of motion result from the action

$$I = \int dt \left(\frac{1}{2}q'^2 - \frac{1}{2l}\varsigma^1\varsigma^{1'} - \frac{1}{2l}\varsigma^2\varsigma^{2'} - \frac{g}{2q^2} + \frac{\sqrt{g}}{2lq^2}[\varsigma^1, \varsigma^2] + \frac{3}{cl^2}T_B q^2 - \frac{3}{cl^2}T_F \varsigma^1 q \right). \quad (4.11)$$

Note that $T_{B,F}$, being external sources, are not varied. The partition function computed with the action (4.11) is the generating functional for correlators of the composite operators q^2 and $\varsigma^1 q$.

If we define the $\mathcal{N} = 1$ superfield

$$\Phi(t) = q(t) + \theta\varsigma^1(t), \quad (4.12)$$

and the supercurrent

$$\mathcal{T}(t) = T_F(t) + 2\theta T_B(t), \quad (4.13)$$

the source term in (4.11) can be written as

$$I_s = \frac{3}{2cl^2} \int dt d\theta \Phi^2 \mathcal{T}. \quad (4.14)$$

In the bosonic case, the emergence of the DFF model of conformal quantum mechanics, coupled to an external source, was first shown in [19].

It is clear that a model of conformal quantum mechanics like the DFF model cannot be invariant under the whole Virasoro algebra. However, when it is coupled to an external field of conformal dimension 2 (e. g. , a stress tensor) that transforms appropriately, it

can have such an invariance. The key point is the following observation:¹² Start from the DFF model with action

$$I = \int dt \left(\frac{q'^2}{2} - \frac{g}{2q^2} \right). \quad (4.15)$$

If we generalize the transformations considered by DFF [21], $t = g(\tau)$, $q = Q\sqrt{\dot{g}(\tau)}$, where a dot denotes a derivative with respect to τ , to arbitrary functions $g(\tau)$, (4.15) goes over (modulo total derivatives) into

$$I = \int d\tau \left(\frac{\dot{Q}^2}{2} - \frac{g}{2Q^2} - \frac{Q^2}{4} \{g, \tau\} \right), \quad (4.16)$$

where

$$\{g, \tau\} = \frac{2\ddot{g}\dot{g} - 3\dot{g}^2}{2\dot{g}^2} \quad (4.17)$$

is the Schwarzian derivative. DFF considered only $\text{PSL}(2, \mathbb{R})$ transformations with

$$g(\tau) = \frac{A\tau + B}{C\tau + D}, \quad AD - BC = 1, \quad (4.18)$$

for which the Schwarzian vanishes, so that the action (4.15) remains invariant. One can obtain invariance under the whole Virasoro algebra, corresponding to arbitrary functions $g(\tau)$, by coupling the field q to an external source $T_B(t)$, that transforms like a stress tensor,

$$\left(\frac{dt}{d\tau} \right)^2 T_B(t) = \tilde{T}_B(\tau) - \frac{c}{12} \{t, \tau\}. \quad (4.19)$$

The transformation law of the coupling term $\sim q^2 T_B(t)$ involves then a Schwarzian derivative that can compensate the one in (4.16), so that the action enjoys invariance under the whole Virasoro algebra.

5 De Sitter gravity

In this section we generalize the above considerations to the case of two-dimensional de Sitter gravity. For related work, cf. [43, 44]. Unlike in higher dimensions, the two-dimensional de Sitter algebra $\text{so}(2, 1)$ admits superextensions that do have unitary highest

¹²For simplicity we consider the bosonic case only. The calculation is easily generalized to the supersymmetric model.

weight representations. Nevertheless, to keep things simple, we will consider below the bosonic case only. The inclusion of supersymmetry should be straightforward.

The action is given by the bosonic part of (2.1), where now $\Lambda = -1/l^2$. Adapting the conventions as in appendix A.2, this model can again be formulated as an $\mathfrak{so}(2, 1) \approx \mathfrak{sl}(2, \mathbb{R})$ topological gauge theory [24], with action

$$I = -\frac{\beta}{2} \int \text{Tr}(\phi F), \quad (5.1)$$

where $F = dA + A \wedge A$ denotes the $\mathfrak{sl}(2, \mathbb{R})$ field strength, and ϕ is an $\mathfrak{sl}(2, \mathbb{R})$ valued scalar field. The gravity action is recovered by decomposing the connection A and the scalar ϕ according to (2.12).

Up to trivial gauge transformations, the boundary conditions for asymptotically ($t \rightarrow \infty$) de Sitter spaces given in [22] read in terms of the gauge fields

$$\begin{aligned} A &= \left[\frac{t}{l^2} - \frac{T_B(\sigma)}{2\eta_0 t} \right] \tau_1 d\sigma + \left[\frac{t}{l^2} + \frac{T_B(\sigma)}{2\eta_0 t} \right] \tau_2 d\sigma + \frac{dt}{t} \tau_0, \\ \phi &= \left[\alpha(\sigma) \frac{t}{l^2} - \frac{\beta(\sigma)}{t} \right] \tau_1 + \left[\alpha(\sigma) \frac{t}{l^2} + \frac{\beta(\sigma)}{t} \right] \tau_2 + \gamma(\sigma) \tau_0, \end{aligned} \quad (5.2)$$

where the functions $T_B(\sigma)$, $\alpha(\sigma)$, $\beta(\sigma)$ and $\gamma(\sigma)$, that depend on the boundary coordinate σ , parametrize the subleading behaviour. The meaning of the constant η_0 will become clear below. Notice that the connection A in (5.2) is flat for arbitrary $T_B(\sigma)$. The metric

$$ds^2 = \frac{l^2}{t^2} dt^2 - \left(\frac{t}{l} - \frac{T_B(\sigma)l}{2\eta_0 t} \right)^2 d\sigma^2, \quad (5.3)$$

resulting from (5.2) satisfies thus the field equations for any $T_B(\sigma)$.

Let us now consider the solutions

$$ds^2 = \left(\frac{t^2}{l^2} - a^2 \right)^{-1} dt^2 - \left(\frac{t^2}{l^2} - a^2 \right) d\sigma^2, \quad (5.4)$$

$$\eta = \eta_0 \frac{t}{l}, \quad (5.5)$$

studied in [44], with a and $\eta_0 \geq 0$ denoting integration constants. In the static patch $0 \leq t \leq al$, t becomes a spacelike coordinate, whereas σ becomes timelike. One can associate to the cosmological event horizon at $t = al$ the mass and entropy [44]

$$M = \frac{1}{2l} \eta_0 a^2, \quad S = 2\pi \eta_0 a. \quad (5.6)$$

Applying the diffeomorphism $t \mapsto t + a^2 l^2 / 4t$ to the metric (5.4) yields

$$ds^2 = \frac{l^2}{t^2} dt^2 - \left(\frac{t}{l} - \frac{a^2 l}{4t} \right)^2 d\sigma^2. \quad (5.7)$$

Comparing this with (5.3), we see that in this case one has $T_B(t) = \eta_0 a^2 / 2 = Ml$, so it essentially coincides with the de Sitter mass.

The gauge transformations

$$\begin{aligned} \delta_\Lambda F &= dF + [F, \Lambda], \\ \delta_\Lambda \phi &= [\phi, \Lambda], \end{aligned} \quad (5.8)$$

that preserve the asymptotic behaviour (5.2) are generated by

$$\begin{aligned} \Lambda^0 &= -\chi'(\sigma), \\ \Lambda^1 &= \left(\frac{t}{l^2} - \frac{T_B(\sigma)}{2\eta_0 t} \right) \chi(\sigma) + \frac{l^2}{2t} \chi''(\sigma), \\ \Lambda^2 &= \left(\frac{t}{l^2} + \frac{T_B(\sigma)}{2\eta_0 t} \right) \chi(\sigma) - \frac{l^2}{2t} \chi''(\sigma). \end{aligned} \quad (5.9)$$

As the function $\chi(\sigma)$ is arbitrary, the asymptotic symmetry algebra is infinite-dimensional. Under a gauge transformation with parameters (5.9), $T_B(\sigma)$ transforms like a stress tensor,

$$\delta T_B = T'_B \chi + 2T_B \chi' - \frac{c}{12} l^2 \chi''', \quad (5.10)$$

with central charge

$$c = 12\eta_0. \quad (5.11)$$

This central charge appearing in the algebra of asymptotic symmetries was found by different methods in [44].

Using the Cardy formula (3.22) with $L_0 = Ml$, and mass M given by Eq. (5.6), we recover precisely the thermodynamical entropy of de Sitter space.

On shell, one finds for the functions $\alpha(\sigma)$, $\beta(\sigma)$ and $\gamma(\sigma)$ that parametrize the subleading behaviour of the scalar field ϕ ,

$$\begin{aligned} T'_B \alpha + 2T_B \alpha' - \frac{c}{12} l^2 \alpha''' &= 0, \\ \frac{c}{6} \beta &= T_B \alpha - \frac{c}{12} l^2 \alpha'', \\ \gamma &= -\alpha'. \end{aligned} \quad (5.12)$$

Again, the dynamics (5.12) of the field α is that of conformal transformations that leave the CFT stress tensor T_B invariant. Like in section 4, defining $\alpha =: q^2$, one finds that (5.12) is equivalent to the equation of motion following from the action

$$I = \int d\sigma \left(\frac{1}{2} q'^2 - \frac{g}{2q^2} + \frac{3}{cl^2} T_B q^2 \right), \quad (5.13)$$

which is the DFF model¹³ coupled to the spin two current T_B .

Let us finally comment on the fact that dS_2 has two conformal boundaries, the past boundary \mathcal{I}^- and the future boundary \mathcal{I}^+ , so in principle one expects a dual description in terms of a quantum mechanical system living on a disconnected manifold. Of course the same problem appears in the AdS_2 case, since two-dimensional anti-de Sitter space, unlike its higher-dimensional cousins, has two boundaries. A possible way to eliminate the second boundary is to cut the spacetime at $r = 0$ ($t = 0$ in the dS_2 case), in order that the position-dependent coupling constant represented by the dilaton (cf. Eqns. (3.4) and (5.5)) is positive [26]. Note in this context that the coordinates used e. g. in (5.4) are not global for $a^2 > 0$, so using this coordinate system we implicitly restricted ourselves to one boundary only.

6 Final remarks

In this paper we investigated the asymptotic dynamics of topological anti-de Sitter supergravity in two dimensions. We showed that the asymptotic symmetries form a super-Virasoro algebra. Using the central charge of this algebra in Cardy's formula, we exactly reproduced the thermodynamical entropy of AdS_2 black holes. Furthermore, we saw that the dynamics of the dilaton and its superpartner reduces to that of superconformal transformations that leave invariant one holomorphic component of the stress tensor supercurrent of a two-dimensional conformal field theory. It was then shown that this dynamics is governed by a supersymmetric extension of the de Alfaro-Fubini-Furlan model of conformal quantum mechanics.

We also considered two-dimensional de Sitter gravity, and computed the dS_2 entropy by counting CFT states. Again, the result coincides with the thermodynamical entropy of dS_2 .

We do not expect that the superconformal structure discovered here is due to the topological nature of the underlying supergravity model. This can be checked by considering more general (non topological) supergravity theories¹⁴ that admit AdS_2 vacua, studying the asymptotic symmetries and determining the dynamics of the fields that parametrize the subleading asymptotic behaviour. Such an analysis should be considerably simplified

¹³Aspects of the DFF model in the context of particle motion in spacetimes containing Killing horizons are studied in [45].

¹⁴Note in this context that also the CGHS model of two-dimensional dilaton gravity can be formulated as a topological gauge theory, based on the centrally extended Poincaré group [46].

using the formulation of two-dimensional dilaton gravity as a Poisson-Sigma-model [47]¹⁵. Work in this direction is in progress.

Acknowledgements

This work was partially supported by INFN, MURST and by the European Commission RTN program HPRN-CT-2000-00131, in which S. C. , D. K. and D. Z. are associated to the University of Torino.

¹⁵See [48] for a review of dilaton gravity in two dimensions.

A Conventions

A.1 Anti-de Sitter

We denote the bosonic $\text{osp}(1|2)$ generators by τ_A , $A = 0, 1, 2$, and the fermionic ones by R^i , $i = 1, 2$. The τ_A satisfy the $\text{sl}(2, \mathbb{R})$ commutation relations

$$[\tau_A, \tau_B] = -\epsilon_{ABC} \tau^C, \quad (\text{A.1})$$

where $\epsilon_{012} = 1$, and indices are raised and lowered with the metric $(\eta_{AB}) = \text{diag}(1, -1, -1)$. Furthermore one has

$$[\tau_A, R^i] = f_A^i{}_j R^j, \quad \{R^i, R^j\} = f^{ij}{}_A \tau^A, \quad (\text{A.2})$$

with $f_A^i{}_j = -(\tau_A)^i{}_j$ and $f^{ij}{}_A = \epsilon^{ik}(\tau_A)^j{}_k$, $\epsilon^{12} = 1$. We choose the explicit representation

$$\tau_0 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \tau_1 = -\frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \frac{1}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (\text{A.3})$$

of the $\text{sl}(2, \mathbb{R})$ generators.

The superalgebra $\text{osp}(1|2)$ admits an invariant, supersymmetric, nondegenerate bilinear form STr defined by

$$\text{STr}(\tau_A \tau_B) = -\frac{1}{2} \eta_{AB}, \quad \text{STr}(R^i R^j) = -\frac{1}{2} \epsilon^{ij}. \quad (\text{A.4})$$

For the gamma matrices in two dimensions we take $\gamma^a = 2i\tau^a$, where $a = 0, 1$. They satisfy

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \quad (\text{A.5})$$

with $(\eta^{ab}) = \text{diag}(1, -1)$. Furthermore we defined $\gamma^{ab} = \frac{1}{2}[\gamma^a, \gamma^b]$, and the parity matrix $\gamma_5 = \gamma_0 \gamma_1$.

Finally, $dt \wedge dr$ is chosen to have positive orientation.

A.2 De Sitter

In order to cover the case of de Sitter gravity, we choose the $\text{sl}(2, \mathbb{R})$ generators τ_A such that

$$\text{Tr}(\tau_A \tau_B) = \frac{1}{2} \eta_{AB}, \quad (\text{A.6})$$

where now $(\eta_{AB}) = \text{diag}(1, -1, 1)$. A possible representation is

$$\tau_0 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \tau_1 = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \tau_2 = -\frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (\text{A.7})$$

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